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FUNDAMENTALS OF QUANTITATIVE ANALYSIS

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Dedicated to my students.

1 Hypothesis Testing

1.1 Hypothesis Testing - Introductory Remarks

The chief reason for misconception about statistics is that one of the central pillars of (frequentist) statistical inference may contradict the fundamental principals of mathematics. Ever since a child starts counting, she or he knows that 1 is not equal 2 or vice versa. Yet, as the time advances and so do the years of schooling, some of the students are exposed to statistics. And for statisticians asking questions seemingly contradicting logic is an everyday practice. This practice is called hypothesis testing, and it actually utterly rely on logic, not contradicts it.

The purpose of testing is not to prove the hypothesis but to estimate the probability that the null hypothesis holds (assuming only the distribution of test statistic is known). If we test a null that the actual average margin is not smaller than the margin planned by the company's board, we do not prove that the sales reps have achieved the plan or they have failed. We merely obtain a probability that the actual margin has not been smaller than the planned one. Test results will always leave plenty of room for interpretation, and unlike the textbook examples, the prosaic business practice produces scenarios with no clear-cut endings. Estimated p-values are not always 0.000 or 0.999. Boarder-line significance is a routine outcome, which produces more questions.

1.2 Defining the Question: Null Hypothesis

The opening step in

In hypothesis testing

A null hypothesis is a research hypothesis, whilst an alternative hypothesis is the converse of the null hypothesis.

A two-tailed test: the statement presented in the null hypothesis concerns equality (=). A one-tailed test: the statement presented in the null hypothesis concerns inequality (strict and weak).

1.3 Answering the Question: Rejection Region and Significance

Rejection region and test value.

Rejection region is a value for which the tested null hypothesis is rejected . For more than a century three levels of significance dominated the testing practice: 0.01, 0.05, and 0.1. Let's assume that the test statistic has a standard normal distribution and that it is a two-tailed test. How to find the value for which the rejection region starts? For a two-tailed test we need to 'cut' 5% from both tails of the distribution, which gives -1.282 and 1.282. If the test value is positive and larger than 1.828, or negative and smaller than -1.828, the null hypothesis is rejected at 10% significance level. Otherwise, the null is not rejected at 10% significance level.

1.4 *p-value*

In order to reject (or not) the null hypothesis we have employed the concept of significance level and the rejection region. The latter, however, has been defined rather arbitrarily. If the distribution of the test statistics is known, another concept might be employed to nullify the research hypothesis. Comparing the test value with the critical value we have operated on the horizontal axis.

The p-value is the empirical probability obtained for the estimated test statistic. In other words, it is an estimated level of significance

Empirical probability

1.5 *Single-Variable Tests*

Planning and performance evaluation are two important aspects of managing a company, by no means unique only to the big corporate world. With a sufficiently large sample at hand, an analyst is able to compare the plan with the actual figures. Testing procedures combine the deviation of actual average from the aim, standard deviation as well as the sample size.

Example 1.5.1. — *The board of Wernham Hogg Co. has set the desired average profit per sales representative at the £15 000 level. For 30 sales representatives in the UK, an average of £14 500 with an astonishingly huge standard deviation of £5 000 has been observed. Have the sales representatives fulfiled the board's aim?*

In terms of the null hypothesis, the problem is re-defined as H_0 : 'the actual average of £14 500 equal to the aim of £15 000'. The estimated p-value is equal to 0.588, and hence the probability that the aim has indeed been obtained is 58.8%.

Example 1.5.1 showes that single-variable tests may help to evaluate the business performance. Many companies set aims in advance (in B2B an average gross margin or desired increase of market share), if the aim can be quantified and the data is available, simple testing procedures offer a tool for evaluating the difference between aims and actual figures.

1.5.1 *Single-Variable Tests: Selected Procedures*

The sample mean and variance of random variable X are $\bar{X}$ and σ_X^2, respectively. The test statistic for a null hypothesis

If X is a time series, make sure the series is stationary.

$$H_0 : \bar{X} = \mu_X,$$

is:

$$t = \frac{\bar{X} - \mu_X}{\sigma_X / \sqrt{T}}, \tag{1.1}$$

and has a Student-t distribution with $T-1$ degrees of freedom. The test statistic for 1.5.1 was estimated using (1.1) and was

$$(14500-1500)\,/5000\sqrt{30}=-0.548.$$

1.6 *Two-Variables Tests*

	Defence	Offence
mean	1.79	2.34
median	0.78	0.65
st.dev.	2.8	4.22
max	15	23.94
T	45	46

Table 1.1: Washington Redskins salaries in $ m for season 2017. Source: http://www.spotrac.com/nfl/washington-redskins/

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